Ways of talking and ways of positioning: Students’ beliefs in an inquiry-oriented differential equations class

Mi-Kyung Ju\textsuperscript{a,1}, Oh Nam Kwon\textsuperscript{b,*}

\textsuperscript{a} Hanyang University, South Korea
\textsuperscript{b} Seoul National University, South Korea

Abstract

As part of developmental research for an inquiry-oriented differential equations course, this study investigates the change in students’ beliefs about mathematics. The discourse analysis has identified two different types of perspective modes – i.e., discourse of the third-person perspective and discourse of the first-person perspective – in the students’ mathematical narratives, depending on their ways of positioning themselves with respect to mathematics. In the third-person perspective discourse, the students positioned themselves as passive recipients of mathematics that has been established by some external authority. In the first-person perspective discourse, the students positioned themselves as active mathematical inquirers and produced mathematics by interweaving their own mathematical ideas and experiences. Over the semester, students’ mathematical discourse changed from third-person perspective narratives to first-person perspective narratives. This change in their discourse pattern is interpreted as an indication of change in their beliefs about mathematics. Finally, this article discusses the instructional features that promote the change.

Keywords: Inquiry-oriented differential equations (IO-DE); Students beliefs of mathematics

1. Introduction

As part of our developmental research in a university-level inquiry-oriented differential equations (IO-DE) course, this article addresses how effectively an inquiry-oriented approach can accommodate the urgent need for reform of mathematics education. In general, our developmental research is situated within the reform movement in the teaching of differential equations stimulated in the mid-1980s by the increased availability of technology and by calculus reform. Our developmental research goes a step further by introducing the inquiry-oriented instruction model to the teaching and learning of differential equations. The IO-DE course is distinct from conventional differential equations classes, which most often deal with the derivation of formulas to find exact solutions of various types of differential equations. The IO-DE course integrates graphical, numerical, and qualitative mathematical methods as well as analytic methods. Students actively participate in the exploration of contextualized problems to construct mathematics through interaction with peers in class. Instead of giving direct instruction, a teacher in the IO-DE orchestrates and facilitates students’ mathematical investigations.

* Corresponding author at: Seoul National University, Department of Mathematics Education, San 56-1, Sillim-dong, Gwanak-gu, Seoul 151-742, South Korea. Tel.: +82 2 880 7894; fax: +82 2 889 1747.

E-mail addresses: mkju11@hanyang.ac.kr (M.-K. Ju), onkwon@snu.ac.kr (O.N. Kwon).

\textsuperscript{1} Hanyang University, Department of Mathematics Education, 17 Haengdang-dong, Seongdong-gu, Seoul 617-736, South Korea. Tel: +82 2 2220 2632; fax: +82 2 2220 4100.

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Since 2000 we have conducted teaching experiments and collected data in order to assess multiple aspects of teaching and learning in IO-DE courses. The evaluation studies have provided evidence to support the positive effect of the IO-DE approach on students’ conceptual understanding, problem solving, retention, and justification (Cho, 2003; Ju & Kwon, 2004; Kim, 2006; Kwon, Cho, Ju, & Shin 2004; Kwon, Park, Kim, Ju, & Shin, 2004b; Kwon, Rasmussen, & Allen, 2005; Rasmussen, Kwon, Allen, Marrongelle, & Burtch, 2006). This study extends our evaluation of the IO-DE course model beyond the cognitive aspects of mathematics learning and investigates changes both in students’ beliefs about mathematics as well as their relation to the discipline.

2. Theoretical background

In recent decades we have seen increasing emphasis on students’ beliefs in educational practice and in mathematics education research (Leder, Pehkonen, & Törner, 2002; McLeod & Adams, 1989; NCTM, 1989, 2000). One important area of research on students’ beliefs came out of studies of gender-related differences. In the 1970s, researchers investigated gender differences in mathematics and found that males and females developed different kinds of beliefs about mathematics, which in turn had subsequent impact on their achievement in mathematics and related careers (Fennema, 1985; Fennema & Peterson, 1985; Fennema & Sherman, 1978; Kloosterman, 1990; Leder, 1982). More substantial progress in research on students’ beliefs about mathematics was made concomitant with the theoretical development of problem solving in the 1980s. In his seminal work on problem solving, Schoenfeld (1985) proposed “beliefs about mathematics” as one of the key components of successful problem solving. Follow-up research has identified a set of beliefs held by students that were shown to have either detrimental or positive consequences in learning mathematics (Carlson, 1999; Kloosterman, 2002; Lampert, 1990; Presmeg, 2002; Schoenfeld, 1989; Stodolsky, 1985). Cobb (1985) observed radically different behaviors among a group of students who were judged to hold similar mathematical concepts. Thus, he argued that since mathematical activity cannot be accounted for solely by mathematical conceptions, more was needed — an analysis of students’ belief systems.

In the early stages of mathematics education research, beliefs were considered psychological traits of individuals, but recent research, based on sociocultural perspectives, highlights the reflexive relationship between an individual’s beliefs and communal norms. Sociocultural perspectives view students’ beliefs about mathematics as not directly derived from the discipline but rather as constructed through years of socialization through schooling that is situated within broader societal contexts such as educational systems, local and national cultures, and ideology (Boaler & Greeno, 2000; Cobb, Yackel, & Wood, 1989; Greer, Verschaffel, & Corte, 2002; Lampert, 1990; McLeod, 2002; Schoenfeld, 1989; Stevenson, Lee, & Stigler, 1986; Yackel & Rasmussen, 2002). In particular, mathematics instruction is considered as a socialization process, and classroom culture is one of the most critical factors that shape students’ beliefs about mathematics. For instance, Yackel and Rasmussen (2002) coordinated sociological and psychological aspects of teaching and learning to show that classroom social and sociomathematical norms and individual beliefs evolve together as a dynamic system. In this reflexive relation, beliefs can be thought of as an individual’s understandings of the normative expectations shared by a class, or a school, as a community. Social and sociomathematical norms can be thought of as taken-as-shared beliefs that constitute a basis for the mathematics practice in class (Cobb, Yackel, & Wood, 1993).

If a belief system is considered to be a sociocultural construct and classroom culture is viewed as a critical factor in shaping student beliefs, then it is necessary to provide educational intervention by developing a classroom with cultural norms that support the positive development of students’ beliefs about mathematics. Indeed, although it is well known that student beliefs are too robust to change over a short term, several teaching experiments report that students’ mathematical beliefs can change over the course of a teaching experiment (Boaler & Greeno, 2000; Cobb et al., 1989; Yackel & Rasmussen, 2002).

In this context, the purpose of this research is to characterize the effect of an IO-DE course on the development of student beliefs about mathematics, especially students’ beliefs about ways of doing mathematics, their relation to mathematics, and their roles in the classroom practice of mathematics. This research takes an interpretive approach to classroom discourse. The specific questions steering this research are: (1) What are the discourse patterns related to the students’ beliefs about mathematics in the IO-DE course? (2) What do the students’ discourse patterns tell about their beliefs about mathematics? (3) According to the discourse pattern, have students’ beliefs about mathematics changed through participation in the IO-DE course? (4) If so, what does the change look like? Based on the results of
the analysis, we will discuss instructional features of an IO-DE course that promoted such changes in order to provide suggestions for further development of inquiry-oriented instruction in mathematics.

3. Methodology

This research employs a sociocultural perspective to consider whether there is a dialogical relation between an individual’s beliefs and communal norms. Norms and beliefs are reflexively related, so change in beliefs and negotiation of classroom norms are inextricably linked. Students’ beliefs in an IO-DE course are investigated based on an analysis of classroom discourse. The use of language in an authentic context of communication is considered to be an important indicator for depicting students’ mathematical reasoning, under the assumption that “facets of cultural values and beliefs, social institutions and forms, roles and personalities, history and ecology of a community may have to be examined in their bearing on communicative events and patterns” (Hymes, 1974, p. 4). Thus, students’ mathematical beliefs, as psychological correlates of collective norms about how to do mathematics, can be inferred through identifying regularities in ways of talking about mathematics in the authentic context of mathematics practice in class.

3.1. Data collection

Data for this research were collected in an undergraduate differential equations course at a Korean university in Fall 2002. We conducted a classroom teaching experiment to emphasize student participation and especially their active construction of mathematics, under the guidance of a teacher. The IO-DE curriculum developed by Rasmussen (2000) was adapted for this particular research setting. In general, the IO-DE course was designed to provide an environment for constructive learning through student participation, explicit meaning negotiation, invention, discussion, and cooperation by integrating students’ mathematical understandings to attain the formal mathematics (Rasmussen & King, 2000).

For this purpose, the IO-DE course exploited problems within a specific context as starting points from which the intended mathematics could emerge. These problems were not only embedded within the mathematical principles but also borrowed students’ real world experiences. Students worked on these problems in groups of three or four. Whole-class discussion followed small-group discussion to share the mathematical ideas that emerged. This cycle was typically repeated two to three times during a class session. The nature of the small-group work was not limited to simply solving a specific problem; rather, students would often analyze a question, develop reasons to support their thinking, and ultimately discover the principal ideas behind the problem context. In whole-class discussions, students collectively constructed arguments and their emergent ideas grew into key concepts of the course, such as slope fields, phase lines, and bifurcation diagrams.

Since the IO-DE course encouraged students’ negotiation of meaning, the teacher took a role distinct from that of a traditional mathematics teacher. The teacher, who is one of the authors, did not lecture but instead orchestrated students’ investigations. During small-group discussions, the teacher walked among the students to facilitate their interaction and to observe how the students were working on a given task. Based on these observations, the teacher planned whole-class discussion. In whole-class discussion, the teacher invited students to share their mathematical ideas. In the IO-DE course, the teacher was not the only authority to judge students’ contributions. Students listened to their peers’ mathematical ideas and were directly involved in evaluating their presentations by asking questions or offering comments to further their inquiries. When a student’s question was directed to the teacher, the teacher would often redirect the question toward the whole class in order to invite other students’ mathematical thoughts. In this way, the teacher guided students to reinvent mathematics through participation.

The class met twice a week for 16 weeks; each session lasted for 75 minutes. There were 19 students enrolled in the class. The course was at a women’s university so all the participants were female. Most of them were freshmen in the mathematics education department. They had taken introductory calculus before they took the IO-DE course and were concurrently taking linear algebra. All class sessions were video-recorded and transcribed for discourse analysis.

3.2. Data analysis

Our discourse analysis sought a pattern of language use that would reflect a change in the students’ beliefs about mathematics. For this purpose, a salient factor in the classroom discourse data was the switch between different kinds of perspective modes. Since perspective modes can be represented by the types of pronouns used, findings
from pronominal variation analysis provided useful guidance for the analysis of the relationship between the use of language and the positionality of human agency in mathematics practice. Generally, studies of pronominal variation in mathematical communication have shown that pronouns serve to code transactional and interactional functions of speech, such as social positioning, interpersonal power relation, formation of a concept, and communication of generalization (Mühlhäusler & Harré, 1990; Pimm, 1987; Rowland, 1992, 1999). For instance, the use of the first-person plural pronoun, “we,” in mathematics class often has the effect of associating a speaker with a powerful group and creates a context of tacit agreement between the expositor and the audience. Such agreement implies that knowledge given by the speaker is proper and, consequently, the hearer is inhibited from raising opposition (Mühlhäusler & Harré, 1990; Pimm, 1987). Use of the second-person pronoun, “you,” often indicates that a certain generalization has been made by a speaker, while use of the first-person pronoun, “I,” refers to personal feelings and beliefs, or accounts of personal actions. By choosing the impersonal “you” in preference to “I,” a speaker detaches herself from what she is asserting and treats her statement as general (Rowland, 1999). For instance, when a teacher is saying “You differentiate a constant to make it zero,” she describes the general rule of differentiation. Thus, the switch between first- and second-person pronouns marks the alternation between two different knowledge domains: subjective and objective.

As discussed so far, in a mathematics classroom, a community develops certain referential practices to represent the positionality of a speaker within the cultural context of communication. The pronominal variation is one such referential practice in mathematical communication that reveals the dynamics of social positioning in the practice of mathematics. In our work, we have extended the analysis of pronoun use to the types of perspective mode by considering that the subject is rather implicitly understood in Korean language as compared to English. This extension reflects the linguistic characteristics of Korean. Korean is categorized as contextualized and functional because it lacks a linear structure among syntactic components. In Korean, an utterance often lacks structural components or violates the structural order. Thus, interlocution is based on the function within a context of communication. In this regard, a functional approach is more appropriate for the analysis of Korean language than a structural approach. Thus, in our analysis, perspective modes are considered not as entirely grammatical but as functional, in the sense that they reflect to whom a speaker attributes her mathematical utterance — external third parties or the speaker herself. If a speaker attributes a mathematical argument to external third parties such as teachers or textbooks, the discourse is coded as the third-person perspective mode speech. If a speaker attributes a mathematical argument to the findings constructed by the course participant, the discourse is coded as the first-person perspective mode speech. The analysis did not count the second-person perspective mode as a separate category because Korean usage of the second-person pronoun is different from English usage. In Korean, the second-person pronoun is used to denote a relationship of familiarity; it is rarely used in formal contexts such as class. No case of the second-person perspective speech appears in our data.

We began analysis of our data by counting the frequency of use of each pronoun. For counting, we chunked the discourse data by turn taking. In this way, we examined the whole argument of a speaker to determine the source of justification as well as the claim. When an utterance was interrupted by the teacher or a peer but the speaker kept constructing her argument, we counted those pieces of utterances as one unit. For comparison, we chose three sessions early in the course (the 2nd session to the 4th session), three sessions in the middle (the 14th session to the 16th session), and the last three sessions (the 21st session to the 23rd session). For each session, we counted the frequency of each type of perspective mode used. The comparison focused on whole-class discussions because the students more consciously referred to what supported their mathematical arguments in the context of whole-class discussions than in their own small groups. The analysis was also limited to students’ mathematical arguments, excluding cases such as short answers, questions to request information, and non-mathematical discourse. In addition to the first-person perspective and the third-person perspective, there were cases where the perspective was ambiguous, when a student made a claim but did not provide enough justification to identify its source.

4. Findings

Table 1 shows the frequency of each discourse type used during the sampled sessions.

As presented in Table 1, the overall pattern is that the percentage of third-person perspective speech decreased and the percentage of first-person perspective speech increased through the semester. Although some students were already using first-person perspective speech on the first day of instruction, the percentage of first-person perspective speech was lower than in sessions later in the semester. Table 1 shows that the percentage of the first-person perspective speech
Table 1
The frequency of each perspective mode speech

<table>
<thead>
<tr>
<th>Session</th>
<th>First person (# of cases, %)</th>
<th>Third person (# of cases, %)</th>
<th>Ambiguous (# of cases, %)</th>
<th>Number of total participants</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1 (14.18)</td>
<td>4 (57.14)</td>
<td>2 (28.57)</td>
<td>4</td>
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<tr>
<td>3</td>
<td>12 (66.66)</td>
<td>4 (22.22)</td>
<td>2 (11.11)</td>
<td>7</td>
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<tr>
<td>4</td>
<td>4 (50.00)</td>
<td>3 (37.50)</td>
<td>1 (12.50)</td>
<td>4</td>
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<tr>
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<td>1 (12.50)</td>
<td>0 (0.00)</td>
<td>4</td>
</tr>
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<td>14</td>
<td>4 (80.00)</td>
<td>0 (0.00)</td>
<td>1 (20.00)</td>
<td>3</td>
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<td>2 (50.00)</td>
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</tr>
<tr>
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<td>2 (22.22)</td>
<td>6</td>
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<tr>
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<td>0 (0.00)</td>
<td>1 (33.33)</td>
<td>3</td>
</tr>
<tr>
<td>23</td>
<td>12 (75.00)</td>
<td>0 (0.00)</td>
<td>2 (25.00)</td>
<td>6</td>
</tr>
</tbody>
</table>

had already increased by the third session. The third session was distinct from the second session because the tasks in the third session provided the students opportunity to manipulate vectors using graphing software. The students worked with visualization technology and discussed their observations on the computer screen. Before the students worked with graphing software, third-person perspective speech was dominant. First-person perspective speech increased significantly as the students began to use graphing software. It was natural for them to talk in the first person about what they had seen. In addition, tasks in the IO-DE course create opportunities for students to conduct mathematical experiments and to speak in their own voices.

It is also important to note the impact of students’ cognitive resources. Cognitive resources refer to students’ experiential knowledge about mathematical facts and methods that emerged out of their own inquiries in the IO-DE class. In particular, the students became more confident to talk about mathematics as their cognitive resources increased. The second session introduced the concept of differential equations to the students. Most students had a static conception of differential equations as equations made up of derivatives. This conception hindered students from thinking about rate of change when solving differential equations, and prompted them to search for algebraic solutions. At this stage, when they encountered a novel task, students believed they were incapable of creating a way to solve it because they were used to following a way provided by a knowledgeable source.

In contrast, in the 15th session, the students approached novel tasks in a very different way. During the 15th session, the students struggled with the new task of drawing a phase curve for the first time. Instead of relying on textbook knowledge, however, they talked about what spring motion would be like. They used their bodies to imitate spring motion and translated their bodily experience into a graphical plane on their worksheets. Also they did not hesitate to share incomplete work. They listened to other groups and developed connections between diverse ideas. They solved this novel problem through a process of collaborative construction.

This difference between the 2nd session and the 15th session cannot be completely explained by kinds of tasks, technology, or cognitive resource. As students consistently encountered an inquiry-oriented learning environment, their beliefs about mathematics changed. For example, consider the following e-mail from a former IO-DE student, sent well after finishing the course:

“When I started taking classes in college after learning only how to do simple calculations in high school math, I totally couldn’t follow the in-depth proofs my college professors were explaining. I was so frustrated many times. I started questioning my math ability and began to think that majoring in mathematics education might be not the path I’m supposed to take. However, I decided to stick with mathematics education, figuring I just needed to master high school math in order to be a high school teacher.

What gave me an opportunity to gain my confidence back was differential equations class. Through differential equations, I experienced what it really meant to do math, and I regained some of my lost confidence. At the least, I felt like I escaped from the pit of my academic slump and got some results to prove it . . .”

This e-mail excerpt suggests that one critical factor for the change in the student’s academic career can be the change in the students’ beliefs about mathematics, such as “what it really meant to do mathematics,” and that the change may be stable and its effect may extend beyond the IO-DE course. Through participating in the IO-DE course, the students
experienced mathematics in a different way than they had been used to in conventional mathematics classes. The students became confident in their ability to do mathematics. They spoke about mathematics in their own voices. In this context, the frequency of the first-person perspective speech increased.

The following two sections illustrate third-person perspective discourse and first-person perspective discourse. The episodes describe the students’ beliefs about mathematics as reflected in the use of each type of perspective mode. The names used are pseudonyms.

4.1. Third-person perspective speech

The following episode illustrates third-person perspective speech as students compared an exact solution with a numerical solution for a given differential equation \( \frac{dP}{dt} = 0.2P \). The students remembered that one of their classmates presented the way to get the exact solution for an identical type of autonomous differential equation in the previous session. Jung-Ah applied the formula to find the exact solution and Eun-Mi asked Jung-Ah to justify the solution. Sun-Ju supported Jung-Ah’s claim by referring to the formula.

(Small-group discussion in the fourth session)

Jung-Ah: It is \( e \) to the zero point two \( t \).
Eun-Mi: Why did we do it this way?
Sun-Ju: That’s what the formula says... to write with \( e \)...
Eun-Mi: Ah...

In this example, Sun-Ju talked about mathematics in the third-person perspective. In her narrative, the subject was “the formula,” that is, an external entity to Sun-Ju. Although Sun-Ju spoke, what justified the students’ mathematical argument was “the formula.”

In this third-person perspective speech, mathematics was represented as a self-contained system of knowledge in the sense that mathematics justifies itself and the students were passive consumers of mathematics. In the beginning of the semester, students typically relied on authoritative sources such as formulas, textbooks, knowledgeable peers, and the mathematics teacher to support their mathematical arguments. This tendency was reflected in the students’ use of third-person perspective narratives:

(Whole-class discussion in the second session)

Teacher: What is the meaning of this equation?
Jin-Hee: I don’t know because I did not preview the textbook.

(Whole-class discussion in the fifth session)

Young-Mi: Uh... before... when we learned about this equation... Professor derived an equation of \( P \) from \( \frac{dP}{dt} \).

In the above examples from whole-class discussion early in the semester, the students often relied on, and accepted, mathematical knowledge given by the textbook or the teacher, even though they did not understand the principle behind the knowledge. Mathematics was considered as a discipline that exists out there, for instance “in a textbook,” independent of their ways of reasoning, and accessible only to special people such as mathematics teachers and gifted mathematicians.

If mathematics is regarded as “a discipline given in a textbook,” the corollary of this belief is that mathematics is an absolute, fixed, and final object for possession. Because of this belief in the certainty of mathematics based on authority, the students rarely questioned the validity or meaning of mathematics.

(Whole-class discussion in the second session)

Young-Ju: My friend told me the method of how to integrate this kind of differential equation. This differential equation contains the derivative of \( x \) with respect to \( t \). Then, the solution will be the integration of this equation with respect to \( t \). Since \( x \) and \( y \) are constant (with respect to \( t \)), you just put \( t \) here.
In the above episode, the task was to identify which represented a predator, \(x\) or \(y\), in a set of given differential equations. The given differential equations could not be solved analytically at this point. However, the student applied the analytic method that some knowledgeable person had told her. Although \(x\) and \(y\) represented populations of certain species along time, that is, functions of \(t\), the student did not care about the meaning of \(x\) and \(y\) and integrated the function as if one of them was an independent variable and the other was a constant. Since a knowledgeable person told her that it was true, the student did not feel the necessity to validate the method or investigate its meaning further. Thus, mathematics was regarded as a set of fixed tools. The students positioned themselves as passive consumers of ready-made mathematics and denied their competence and right to interpret and produce mathematics.

These beliefs about mathematics became obstacles to students’ participation in the inquiry-oriented practice of mathematics in the IO-DE class. However, this obstacle did not, in fact, restrict the ability of the students to grow in their learning; it actually created a space for problematizing the students’ experience of mathematics and for negotiating their beliefs about mathematics. This belief negotiation could be detected by the students’ use of the first-person perspective mode narratives in mathematical arguments later on in the semester.

4.2. First-person perspective speech

First-person perspective speech refers to discourse whose subject is the students themselves, either an individual or the group that a speaker belongs to. Discourse analysis shows that the students’ mathematical discourse increasingly employed the first-person perspective mode of speech through the course of the semester. When the first-person perspective narratives reflect qualitatively different beliefs about mathematics, the transformation in the perspective mode can be interpreted as evidence to support the change in the students’ beliefs about mathematics. The most salient difference is concerned with the notion of agency, that is, who is thinking, speaking, and doing mathematics. The third-person perspective discourse revealed the students’ beliefs concerning mathematical agency as external to them. In contrast, in the first-person perspective discourse, the students position themselves as active agents of doing mathematics.

(Whole-class discussion in the 14th session)

Sun-Hee: We computed the average values for the data from the table. We connected the points on the plane and noticed that the rate of change was decreasing. The rate of change became almost zero near twenty-one. So we thought that the solution space has an equilibrium solution at twenty-one and the curves decrease near the equilibrium solution.

In this excerpt, the student described how her group modeled the temperature of cooling coffee and what the solution space looked like. Sun-Hee presented the method and the conclusion of her group with confidence, as evidenced by her use of “we;” which included the members of her group. She did not refer to outside authority. She talked about how her group approached the task. In this episode, the student spoke in the first-person discourse, which revealed her sense of ownership of mathematics. She positioned her group, including herself, as an active agent in the practice of mathematics and such positonality is expressed by the use of the first-person perspective discourse. In addition, by introducing what her group observed into her mathematical argument, Sun-Hee suggested that mathematics emerged through collective inquiry by her group instead of being given to them as a finished system.

The following conversation is from the 15th session in which the students explored spring motion and drew a graph to describe the motion in a phase plane. The task was for students to represent velocity with respect to position. Because the students were accustomed to thinking about motion with respect to time, this task was novel and challenging. The whole class struggled with this task but their efforts were not being fruitful. One group completed a graph but the graph was the conventional time–distance graph. The teacher invited the group to share:

(Whole-class discussion in the 15th session)

Hee-Young: Uhm... How did I begin (Talking to herself)? Ahha. I thought that it would be difficult to draw a graph of velocity with respect to position... \(x-t\) graph would look nicer. So I determined to begin with \(x-t\) because I guessed that the graph \(x-t\) would be a good place to start.

When Hee-Young began this presentation, she knew that her group’s solution was not perfect. However, in her presentation, Hee-Young tried to describe her own reasoning process of how to construct the phase curve in detail.
Even though the solution was incomplete, she argued that her graph was constructed reasonably according to the conventional method. In this context, the use of the first person can be seen as revealing the student’s beliefs concerning the ownership of mathematics. This represents a remarkable change from the students’ attitudes at the beginning of the semester, when they regarded only perfect solutions as desirable and their presentations dealt only with the final product of exploration.

Moreover, this episode shows the change in Hee-Young’s understanding of norms regarding what mathematics is worthy of discussion, who does mathematical knowledge belong to, and how, in general, to do mathematics. Indeed, although Hee-Young did not accomplish what the task demanded, her presentation was productive because it provoked inspiration in her peers. After this presentation, another student, Jung-Sook, commented:

(Whole-class discussion in the 15th session)

Jung-Sook: I looked at the graph that Hee-Young drew and thought... that the duration between zeros of $x$ decreases and the duration between zeros of $v$ decreases too. So I thought that the graph of $v$ to $x$ would look like a spiral.

In this excerpt, Jung-Sook also justified her graph based on her observations of her classmate’s work. Jung-Sook spoke in the first person to describe the process of how she reached her graph. Instead of singling out the final product, Jung-Sook situated her reasoning within the context of communication to highlight how mathematics emerged through co-engagement. This suggests that in the IO-DE course, the students began to appreciate their own competence to produce mathematics and to develop an evolving perspective on mathematics through co-engagement instead of solitary reasoning.

4.3. Ways of talking about mathematics and beliefs about mathematics

So far, we have illustrated two different types of perspective modes used by the IO-DE students to describe their beliefs about mathematics. The discourse analysis shows that the students’ speech patterns changed from the third-person perspective mode to the first-person perspective mode during their participation in the IO-DE class. We argued that this change paralleled the transformation of the students’ beliefs about mathematics, in particular what they perceived as a proper way of doing mathematics, the nature of the discipline, the foundation of mathematical certainty, and their views of human agency in the practice of mathematics. The third-person perspective discourse reflects the students’ beliefs about mathematics as a set of absolute and immutable truths independent of what the students think. From this perspective, it is natural that the students positioned themselves as passive recipients or consumers of mathematics. In the first-person perspective discourse, the students represented themselves as active agents in producing mathematics. They came to see their experience, such as observation, as a valid source of mathematical argument. Thus, mathematics was considered as deeply situated within the lived experience of the students. This change led the students to realize how it was possible to approach mathematics from different angles and how important it was for the productive practice of mathematics to admit such differences. In the IO-DE course, the students came to respect each participant’s mathematical expertise and to appreciate mathematics as emergent through co-engagement with peers who have different kinds of expertise.

The IO-DE students’ use of the different types of perspective mode revealed the different ways that the students positioned themselves in the practice of mathematics. This pattern is comparable to the findings from research on professional scientists’ everyday discourse. Ochs and her colleagues analyzed the discourse of physicists working at a laboratory and identified patterns of referential practices such as “physics-centered discourse” and “physicist-centered discourse” (Ochs, Gonzales, & Jacoby, 1994). Physics-centered discourse is a type of discourse in which physical entities are the agents (e.g., “a particle crosses to a domain state”; Ochs, Gonzales, & Jacoby, 1996, p. 337). In this type of discourse, scientists focus on inanimate physical entities and act as observers of events. In contrast, in physicist-centered discourse (e.g., “If you cool extremely fast, the system may never have the time to experience those random fields”; Ochs et al., 1996, p. 338), scientists refer to themselves as the agents and experimenters of physical phenomena. In this way, scientists position themselves as active participants in the making of science and scientific discovery. Moreover, in physicist-centered discourse, physicists become subjectively engaged with physics as experiencing the physical world. For instance, when a scientist says “When I come down, I’m in the domain state” (Ochs et al., 1996, p. 343), she puts herself in the position of a particle as a scientific object. That is, scientists and science are treated as
blended thematic foci and this indeterminacy blurs the distinction between scientists and physics and reveals extreme subjective involvement with science.

The first-person perspective mode discourse in the IO-DE resembled physicist-centered discourse (Ochs et al., 1994, 1996). In first-person perspective mode narratives, the students characterized themselves as agents actively involved in the construction of mathematics. As the students gradually switched from third person to first person, the students interwove their mathematical ideas and previously shared mathematical meanings into their practice of mathematics. Through this process, they came to view themselves as active producers of mathematics and mathematics as a human practice that was constructed collectively through their own engagement. This is similar to the views held by professional mathematicians (Burton, 1995).

5. Discussion

Our analysis of students’ use of language in the IO-DE course has identified its contribution to the positive transformation of their beliefs about mathematics. Although it is well known that beliefs are too robust to change over a short term, there are studies reporting that students’ mathematical beliefs change dramatically over the course of a longer teaching experiment (e.g., Cobb et al., 1989; Yackel & Rasmussen, 2002). These studies suggest that a mathematics classroom can be considered to be a community of practice with certain norms of doing mathematics and that those classroom norms and students’ beliefs emerge in a dialogical relation. It is important, then, that teachers deliberately initiate the negotiation of classroom norms to promote the positive development of students’ mathematical beliefs. The following section discusses the instructional features of the further educational intervention that facilitated the change in students’ beliefs about mathematics in the IO-DE course.

5.1. Instructional features of IO-DE

Because the IO-DE class differed from the conventional mathematics classes in so many ways, the teacher began the course by carefully explaining expectations for students, such as being active participants and listening to, respecting, and thoughtfully challenging peers’ mathematical ideas. The teacher reminded students of the purpose of these expectations — to facilitate students’ interaction in class. Although the explicit communication of course expectations might have influenced the students’ belief systems, the analysis shows that the students’ beliefs about mathematics only came to change through their participation in the daily practice of mathematics in the IO-DE class. As discussed, the mathematics class is a cultural arena and students became transformed within this arena. Thus, an inquiry into the cultural organization of the IO-DE has great significance for the evaluation of the course and, more importantly, for our deeper understanding the change in the IO-DE students’ beliefs about mathematics. What follows now is a discussion of three instructional features that contributed to the positive transformation of students’ beliefs about mathematics: instructional materials, students’ cognitive resources, and the role of the teacher.

5.1.1. Instructional materials

The instructional materials used in the IO-DE course contributed to the positive transformation of the students’ beliefs about mathematics. The materials included the use of context problems and technology. Experientially real context problems helped the students pull informal mathematical understanding out of their lived world and integrate it into their mathematical inquiry.

(Whole-class discussion in the second session)

So-Hyung: Does the population of the predators always increase? We learned that in the beginning, the population of the predators increases exponentially because of the plentiful resources. However, the predators eventually starve and are extinguished. The population decreases, so I guess the rate of change becomes negative.

In this episode, the student thought about the relation between the rate of change and the predator population change over time. In her argument, she introduced her knowledge that the predator population would increase but begin to decrease when the predators had eaten all the prey. Since this problem was constructed around a context that the students were familiar with, the students often turned to their knowledge of their experience in the lived world and
extended it to their mathematical investigation. In addition, the fact that this episode came from the second session tells us that the transformation in the students’ belief system had begun very early in the semester.

The IO-DE materials used technology such as graphing calculators and java applets. These tools supported mathematical inquiry by providing visualization of the mathematical contexts under inquiry; this enabled the students to talk about mathematics based on “what they observed.”

(Whole-class discussion in the 17th session)

Nami: When n is 1, we checked on the graph and found there was no straight line solution. We thought that there was no straight line solution because the determinant became negative.

In her small-group discussion, Nami used a java applet to try several values. Through observation, her group came to find a pattern that the number of straight line solutions depended on the sign of the determinant and reported this finding in the first-person perspective discourse.

5.1.2. Cognitive resources

The emphasis on students’ own cognitive resources – students’ own experiential knowledge that they have accumulated about mathematical facts and methods – is another instructional feature of the IO-DE class which influences students’ beliefs about mathematics in a positive way.

(Small-group discussion in the fourth session)

Soo-Jung: Put zero point five into t.

Mi-Ju: I don't think that would work. I tried that before but the same result comes out.

La-Mi: On the graphing calculator, we saw that the graph changed when we changed the time unit. Ah. Don’t you remember that the graph did not change when we preserved the ratio between the coefficient and the time unit?

Ji-Hyun: You’re right.

The IO-DE course highly values students’ justification; their mathematical experience becomes interwoven into the process of constructing a mathematical solution. In general, the students are encouraged to build their own mathematics through sense-making, instead of simply acquiring textbook-like knowledge. In the above example, the students collaborated to solve a problem by sharing previous mathematical experiences in the course. When Soo-Jung offered an idea, Mi-Ju mentioned that her previous experience with that method was not fruitful. La-Mi reminded them what they had experienced with a graphing calculator during the previous session. In the previous session, the class worked with a graphing calculator to draw a slope field given by a differential equation. In the graphing calculator, a user defines the time duration for a slope mark. The students experimented with the graphing calculator and one of their experiments involved changing the time duration to observe how the overall picture of a slope field changed with respect to time duration and how a slope field could be reproduced when the time duration changed. La-Mi mentioned the experiential knowledge she gained from this activity: to preserve the ratio between the coefficient and the time duration. Since the cognitive resource was accumulated through sense-making, it was easily retrieved and integrated into the students’ mathematical investigation. Furthermore, cognitive resources enabled the students to approach a mathematical context from their own mathematical perspective. So, the more cognitive resources accumulated, the more likely students were to rely upon their own cognitive resources to enact their roles as active agents of mathematics.

5.1.3. Role of the teacher

The data also show that the mathematics teacher plays a critical role in the positive transformation of the students’ beliefs about mathematics. Previous research about beliefs has drawn attention to the link between a teacher’s beliefs about mathematics and her educational practice in the mathematics class (e.g., Leder et al., 2002; Thompson, 1992). As described earlier, the IO-DE course is different from conventional Korean mathematics classes. Since students’ beliefs about mathematics are the product of the cultural organization of a mathematics class, it was anticipated that the students would experience initial conflict and confusion due to the discrepancy between the IO-DE course expectations and the students’ previous experience in school. However, the conflict and confusion were a productive starting point
for negotiation, especially with the teacher’s guidance. For instance, throughout the semester, the teacher encouraged a way of doing mathematics that challenged what the students had believed was appropriate:

(Whole-class discussion in the third session)

Teacher: Nobody knows the perfect solution for the problem in this class. This is not about finding a formula. You just think about things like whether your peers’ ideas make sense, whether their ideas are logically appropriate, whether your reasoning is logically valid in this problem context, and so on.

In the previous conversation, the class thought about the task about population growth so as to determine whether it was an initial condition or time which influenced the phenomenon of change. In the whole-class discussion, the teacher invited the students to present their work. The students seemed reluctant to talk about their ideas because their answers might not be correct. As an experienced practitioner of mathematics, the teacher knew that the ability to construct a valid argument is as significant as the ability to set up a formula. The teacher communicated this perspective and challenged what the students had believed to be proper in doing mathematics. The teacher has developed a belief system about mathematics through her long engagement with mathematics. In the above episode, the teacher explicitly communicates her beliefs about mathematics by stating what is worthy of discussion. At the same time, the teacher implicitly delivers her beliefs system to the students through her practice of mathematics in the IO-DE class. In the class, it was observed that the teacher adopted the first-person perspective discourse. In demonstrating another way of doing mathematics, the teacher provided an opportunity for the students to consider how they might do mathematics in the IO-DE class.

(Whole-class discussion in the fourth session)

Teacher: Can we elaborate on what Eunjoo said in her presentation? What our friend Eunjoo said was that we constructed an approximate solution by using tangent vectors. She said that they could make a better approximation by making the interval smaller. Can we extend this idea to develop a way to get an exact solution?

In this excerpt, it is important to note the way the teacher used the first-person plural pronoun. Pimm (1987) argued that mathematics teachers use the pronoun “we” to authorize an argument by appealing to an anonymous expert community outside the class. For example, when a mathematics teacher says, “What do we take from the tens column? We take a ten, don’t we?” the teacher intends to remind students of a computational procedure. In this context, the use of “we” is a strategy to persuade students to accept the procedure by communicating that this procedure is already established as shared. This type of discursive practice characterizes mathematics as a mysterious system of absolute truths. The IO-DE teacher’s use of “we” in the previous episode contrasts to this exclusive use of “we”; this “we” refers to all the participants in the class. Although the presented method had been established as shared in a community of professional mathematicians, the teacher valued the fact that the method had emerged and been refined through the students’ own mathematical practice. This inclusive use of “we” in the IO-DE class developed a sense of authorship and ownership of the knowledge among the students.

5.2. IO-DE: deconstruct to reconstruct the reality of mathematics class

In modern society, school reform has worked as a driver of economic revival and national prominence. There has been general euphoria about the role of mathematics in social and material improvement. However, continual pressure toward economic transformation leads to an emphasis on the technical aspect of mathematics in education, which has enhanced the status of professionalized knowledge in the narrowest sense. Mathematics has been treated as a commodity that generates a return for learners. Thus, teaching practice has focused on the technical aspect of the discipline in order to supply a well-equipped labor force to the market (Apple, 1992; Pospewitz, 1991; Stanic, 1986). Mathematics classrooms are not insulated from the communities in which they exist; rather, they are constituted by the external community’s interests and agenda. Thus, in mathematics class, a student encounters the discipline situated within the complex value system of a larger society. Thus the students’ learning of mathematics is constrained by a certain kind of belief system about mathematics, which forms a student’s values and norms about mathematics. For example, Boaler and Greeno (2000) compared two different types of mathematics classrooms based on didactic teaching and discussion-based teaching. They found that these two different ways of teaching mathematics were concerned with different ways of positioning learners in the practice of mathematics and led to the development of different types
of identity. Specifically, in the didactic class, students defined themselves as passive recipients of mathematics from a teacher and textbooks. These students prioritized willingness to accept a particular form of knowledge given by authority and saw human agency as playing a minimal role in success in mathematics class. Although students are one of the agents, the trajectory of students’ transformation is dominated by the culture of the mathematics class, especially when students are unaware of their own agency. It is thus important to raise a student’s awareness of agency. Our study suggests the importance of the teacher’s role in this regard. Specifically, a mathematics teacher should play the role of guide by critically reflecting upon socially shared values and norms in which the practice of school mathematics is situated, and by revealing for students the boundaries of the current practice of mathematics.

Therefore, one of the most prominent features of the IO-DE course is the notion of decentralizing mathematics practice. Traditionally, a mathematics teacher has been the center of the practice in mathematics class. Teaching and learning mathematics has been based on the traditional view of mathematics that only the mathematics teacher possesses a complete system of mathematics, to be transmitted to students through instruction. It is this view of mathematics that confers on a mathematics teacher the authority to control knowledge production in class and legitimize a certain kind of mathematics. As a consequence, it was only the teacher’s voice that was heard in mathematics class. Students’ voices were neglected because they were considered mathematically incompetent. In contrast, the IO-DE course decentralizes the mathematics practice in class and invites more than one way of knowing. The emphasis on students’ own construction of mathematics implies respect for the students’ practice of mathematics. The teacher had to consider all the logical contingencies and reconstruct the goals of learning in the context of students’ mathematical practice. Thus, the teacher was not in charge of learning in the IO-DE class but co-engaged with students in practice. Authority over knowledge production was distributed among the participants in the IO-DE class. Truth was not based on who said it but rather on how it was built.

The decentralized structure of the IO-DE course presented a challenge not only to the students but also to the teachers. However, the challenge is not merely to deconstruct but to reconstruct the reality of the mathematics class by problematizing what we have taken as a status quo, and to transcend the boundary of practice in conventional mathematics class. In the IO-DE class, the students learned to think in their own ways and to talk about mathematics in their own voices, and they witnessed how much they could accomplish. The students developed a sense of ownership of the mathematics that they had constructed. The students observed that mathematics emerged through co-engagement by sharing stories of their mathematical investigation. These experiences helped to demystify mathematics for the students and helped them to see mathematics as more than merely a discipline of cool reason. This experience of viewing mathematics and oneself in a different way should be emphasized in teaching and learning mathematics in light of Freire’s dictum (1970) that oppression is ultimately about limiting one’s ways of seeing the world.

In this research, we investigated the effect of IO-DE on students’ mathematical beliefs. We also discussed the instructional features that promoted the positive development of mathematical beliefs. Our research showed that students’ beliefs about mathematics were transformed through their participation in the IO-DE class over relatively short span and that systematic intervention based on the sociomathematical norms of inquiry-oriented mathematics was critical for that purpose.

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References


