# Securely Create Public Keys and Private Keys for Sensor Networks

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Abstract—Due to the potentially unsecured environment of a wireless sensor network, protecting it from any attack is significant importance to maintain the health of the network. Recently, many researchers have focused on providing the security on making sensor networks available and reliable for the nodes to communicate to each other.

In this paper, we propose a secure and efficient method for node to node key agreement in unsecured wireless sensor networks. The main idea of the proposed key agreement is to use a ticketing grant server and a key server to generate a shared key for the principal nodes to communicate in the wireless sensor network environment. To create the secure environment, two cryptography protocols take into our consideration, symmetric ciphers such as shared key encryption and asymmetric ciphers such as public-private key encryption. Wireless sensor networks are resource-constrained environments, which are improper for computationally expensive that occurs in public-private key encryption. Therefore, the key in our protocol is solely based on a symmetric protocol provided by two trusted agents, the ticketing grant server and the key server. A ticketing server is to provide a ticket for the principal nodes to communicate with the key server. Then the key server generates a shared key that principal nodes can use to secure the communication between each other. The confidentiality, authentication, and freshness of the network security are discussed to analyze the performance of our proposed method.

Index Terms—Wireless sensor networks; Security; Authentication; Confidential; Freshness

#### I. INTRODUCTION

Wireless sensor networks (WSN) are composed of a potentially large set of devices that communicate over a wireless medium. Such networks can be formed spontaneously whenever devices are in communication or transmission range. Joining and leaving of nodes occurs dynamically, particularly in the presence of mobility (e.g., vehicular sensor networks). Two mobile devices out of communication range can use intermediary devices for relaying packets. For each area of wireless sensor networks, a cluster of sensor nodes is deployed to collect the data that is later reported to the network base station [1]. To collect the data in such a network, the sensor nodes can relay the data to the base station. Although, many advantages are in favor of the use of wireless sensor networks, they also suffer from limited battery lifetime, interference, noise and—in particular—security. Potential applications of wireless sensor networks can be found in traffic scenarios,

vehicular ad hoc and sensor networks, ubiquitous Internet access, and collaborative work [2], [3], [4].

Due to the nature of these applications which always broadcasts the radio transmission from the sensor nodes to the base station, the wireless sensor network is more vulnerable to network attacks than other traditional networks. In order to protect the sensor nodes from the attacks, the implementation of cryptography becomes more important to achieve the objectives of network security. Several topics of cryptography have been well studied for traditional networks. However, many conventional cryptographic approaches cannot be appropriately applied to the wireless sensor networks. For the implementation analysis, public-private key schemes and some symmetric key methods are still too complicated for the wireless sensor networks in the terms of computations, memory, communications, and packet size requirements. In addition, wireless sensor networks suffer from some other severe constraints on the network resources which refer to the necessity of increasing the network lifetime, minimizing the physical size of the sensor nodes, and reducing the cost of sensor nodes.

A wireless sensor network can be considered as highly intensive data collection (i.e., meaning that the security of data transfer and access becomes critical issues). Three well known security goals are: only authorized user can access to the data (Confidentiality), the data should be genuine (Integrity), and the data should be available for the authorized user (Availability) [5]. All these goals are the requirements from both users and wireless sensor networks.

A possible attack scenario in wireless sensor networks includes physical destruction of sensor nodes, security attacks on the routing and data link protocols, and resource consumption attacks. Unattended sensor node deployment can cause another attack in which an adversary may try to compromise several sensor nodes and inject false data into the network through the compromised sensor nodes.

In this paper, a secured node-to-node key agreement is proposed to ensure that only an authorized user can access the network, and it is ensured that data is only available for the authorized user. The key in our protocol is solely based on a symmetric protocol provided by two trusted agents, the ticketing grant server and the key server. The proposed

protocol is less complex compared to the other protocols that apply asymmetric encryption scheme with regards to the computation, memory usage, and packet size requirements.

The structure of this paper is organized as follows. Section II describes the related work. Section III explains the implementation of the proposed protocol; Section IV presents a performance comparison with existing methods and provides a complexity analysis and Section VI concludes the work of this paper.

#### II. RELATED WORK

Related work that focuses on the security issues in computer networks is presented here.

In [6], Fox and Gribble described the security and authentication on open networks. This protocol provides lightweight secured communication at the client module using the interaction with a proxy to Kerberos [7] at the application-level proxy service.

In [8], Patel and Crowcroft provided the security in mobile user where asymmetric cryptography has been used with a ticket-based service access model allowing anonymous service usage in mobile application. The mobile users anonymously contact the credential center to check for the user's certification for the access. However, the asymmetric cryptography is costinefficient for a sensor network environment.

In [9], Perrig et al. presented a security protocol for multicast communication. The paper focused on securing multicast communication at the source authentication and enabling receivers of multicast data to ensure that the received data is originated from the source. The paper proposed the modification to TESLA (Timed Efficient Stream Loss-tolerant Authentication) to allow receivers to authenticate the packets when they arrive. TESLA implements a loose time synchronization between the sender and the receivers, followed by delayed release of keys by the sender.

In [10], Liu and Ning presented the broadcast authentication in distributed sensor networks where the development of a scalable broadcast authentication scheme named multilevel  $\mu$ TESLA based on the original  $\mu$ TESLA protocol [11]. Multilevel  $\mu$ TESLA provided several improvement which included low overhead, tolerance of message loss, scalability to large networks, resistance to replay attacks, and denial-of-service attacks (DoS).

In [12], Ren et al. claimed that both  $\mu TESLA$  and multi-level  $\mu TESLA$  suffered from serious DoS attacks due to the delay of message authentication. They used the public key concept to achieve the broadcast authentication. Cryptographic techniques including Merkle hash tree and identity-based signature scheme have been adopted to minimize the costs on computation and communication in wireless sensor networks.

In [13], Sultana et al. focused on malicious packet dropping attack in the sensor network. They proposed a data provenance based mechanism to detect the attack and identify the malicious node. The proposed scheme based on the watermarking based secure provenance transmission. The scheme includes

packet loss detection, identification of attack presence, and localizing the malicious node/link.

In [14], Oliveira et al. used Pairing-Based Cryptography (PBC) protocols for key distribution in wireless sensor networks. The concept of Pairing-based Cryptography (PBC) protocols is that the parties can agree on keys without any interaction. The TinyPBC presented in this paper was able to compute pairings, the most expensive primitive of PBC for 8, 16 and 32-bit processors sensor nodes.

#### III. IMPLEMENTATION

### A. Routing protocol

The routing protocol that we have chosen for this paper is SOFROP (Self-Organizing and Fair ROuting Protocol) [4], [15]. This protocol is efficient and provides the lightweight routing that is optimized for fairness and the locally acting adaptive overlay network formation. SOFROP is a routing protocol capable of dealing with an unpredictable environment with a diversity of mobility elements. On the network organization level, it is able to continuously adapt the sensor network topology and deal with challenges such as rapid changes in the link structure. The protocol is proven to provide an efficient bandwidth utilization and data transmission, which are important aspects for the settings used in this paper.

SOFROP considers a topology control layer, which determines dedicated nodes to be the *cluster heads* (or *routing nodes*). These routing nodes form a backbone network on a otherwise flat network topology. The routing nodes election process can be implemented by using heuristics [16] or a multi-hop hierarchical clustering structure (e.g., applying the multi-hop clustering algorithm KHOPCA [17]).

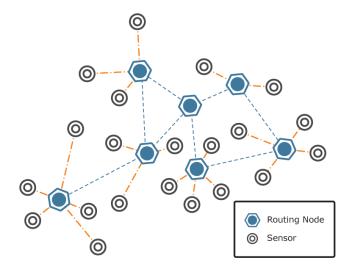


Figure 2. Wireless sensor nodes and routing nodes.

#### B. Node-to-node key agreement

After SOFROP identifies the next sensor node, the identified node needs to communicate with the previous node along the communication path. This means node-to-node key

Step	Key Agreement
1	$A \rightarrow B: N_{A_1}, A$
2	$B \to T: N_{A_1}, N_{B_1}, A, B, MAC(K_B, N_{A_1} N_{B_1} A B)$
3	$T \rightarrow A : \left\{ K_{TSA} \right\}_{k_A}, MAC \left( K'_{TSA}   N_{A_1}   A   B   \left\{ T_n   K_{TSA} \right\}_{k_A} \right)$
4	$T \to B : \{K_{TSB}\}_{k_B}, MAC(K'_{TSB} N_{B_1} A B \{T_n K_{TSB}\}_{k_B})$
5	$A \to S: N_{A_2}, A, B, MAC\left(K'_{TSA} N_{A_2} A B \{T_n\}_{k_{TSA}}\right)$
6	$B \to S: N_{B_2}, A, B, MAC\left(K'_{TSB} N_{B_2} A B \{T_n\}_{k_{TSB}}\right)$
7	$S \to A : \{K_{AB}\}_{k_A}, MAC(K'_A N_{A_2} A B \{K_{AB}\}_{k_A})$
8	$S \to B : \{K_{AB}\}_{k_B}, MAC(K'_B N_{B_1} A B \{K_{AB}\}_{k_B})$

Figure 1. The eight steps of the key agreement.

agreement has to be established. The proposed key agreement in this paper provides the security of the communication which includes authentication, availability, and freshness. Two cryptography protocols take into our consideration. The former protocol is symmetric ciphers such as shared key encryption. This symmetric ciphers require one public key generated for the communication between nodes. The latter protocol is asymmetric ciphers such as public-private key encryption. This asymmetric ciphers required two different key generated.

As mentioned in [11], due to the resource-constrained in the sensor nodes, the computationally expensive public-private key cryptography in asymmetric ciphers is not suitable. Therefore, the protocol for symmetric-key is suggested.

In addition for the purpose of gaining stronger freshness, the nounce numbers are used for the communication between the principal nodes and the two trusted agents, key server S and the ticket granting server T. A message authentication code (MAC) is additionally used for two-party authentication and data integrity.

For the node-to-node key agreement description, we use the following notations in this paper.

A,Bare principles, such as communication nodes  $N_{A_1}, N_{A_2}, N_{B_1}, N_{B_2}$  are Nonce numbers from A and B  $M_1 | M_2$ denotes the concatenation of message  $M_1$  and  $M_2$  $K_A$ is the key of node A shared with base station  $K_{AB}$ is the pairwise key shared between nodes A and  $K_{TSA}$ is the shared key between nodes T, S, and A  $\{M\}_{k_{TSA}}$ is the encrypted message with the symmetric key  $T_n \\ K_{TSA}$ is the ticket number to communicate with Tis the shred key between nodes A and BMACis the message authentication code

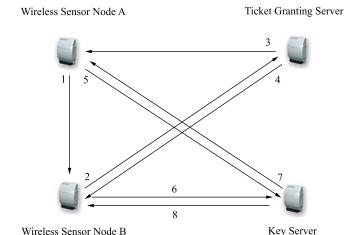


Figure 3. Illustration of the steps of the proposed node-to-node key agreement.

The proposed node-to-node key agreement provides secured key agreement and stronger key freshness. The step of the key agreement is as follows:

In our key agreement  $N_{A_1}, N_{A_2}, N_{B_1}$  and  $N_{B_2}$  that produced by node A and B guarantee the freshness of the communication between principle nodes A and B, ticket granting server 1 and the key server S. The security is increased by not only for connecting to the key server but also to the ticket granting server. To provide the confidentiality, the key  $K_{TAB}$  is used for the communication between nodes A and B. This ticket is authenticated with the message 3authentication code MAC.

The detail of each step of the proposed method is explained here. We start from step 1 when the wireless sensor node A would like to communicate with node B. Node A sends a request to node B with nonce number  $N_{A_1}$ . In step 2 node B sends  $N_{B_1}$  with a request for a ticket from ticket granting

server T. This ticket is used for communication between node A and B, and the key server S. In steps 3 and 4, the ticket granting server T grants a ticket and sends to nodes A and B.

The messages sent from the ticket granting server T to nodes A and B are encrypted with key  $K_A$  and key  $K_B$ , respectively. Then in steps 5 and 6, nodes A and B send the ticket to the key server S to ask for the shared key  $K_{AB}$ . At these steps, nodes A and B sends the messages to the key server with the nonce numbers  $N_{A_2}$  and  $N_{B_2}$  respectively. As mentioned earlier, the nonce numbers provide the freshness of the communication. The key server S receives the ticket and then issues the shared key  $K_{AB}$  to nodes A and B in steps 7 and 8. For the confidentiality, all messages sent from either the ticket granting server T or the key server S to nodes A or B are encrypted by  $K_A$  or  $K_B$ . Thus only node A or node B can decrypt the messages. All these messages are authenticated with MAC protocol. Thus the proposed method provides the three properties of wireless sensor security for node to node key agreement; all of which are freshness, confidential, and authentication.

#### C. MAC Authentication

MAC Authentication is a cryptographic technique that computes checksum on data. This cryptography uses a session key to detect both accidental and intentional modifications of the data. Two inputs, a message and a secret key that is known by only the original sender and its intended recipient, are required for a MAC authentication. This cryptography allows the recipient to verify the integrity of the message and to authenticate the message as only the original sender and its intended recipient have the shared secret key. In case the sender is an attacker who does not know the secret key, the value that is returned form the hash function would be different and that would notify to the recipient that the message was not sent by the original sender. On another hand, if the attacker pretends to be the recipient who does not know the secret key, then the message will not be read by the pretending recipient.

Four types of MACs can be selected; all of which are unconditionally secure, hash function-based, stream cipher-based and block cipher-based. The most common approach to creating a MAC that was common used in the past was the block ciphers such as Data Encryption Standard (DES). However hash-based MACs (HMACs) that apply a secret key in conjunction with a cryptographic hash function to produce a hash, have become more widely used and is used in the proposed scheme.

#### IV. COMPARISON AND COMPUTATION ANALYSIS

To evaluate and compare the performance of the proposed key agreement scheme with the existing schemes for wireless sensor networks, the complexity of communication and procession operations is the important metric. In this manner, our proposed key agreement scheme has some advantages over the existing public and private key schemes. With this proposed method, the high cost public-key encryption has been replaced by symmetric encryption at the sensor nodes with the hash function at the MAC authentication. In addition, to establish node-to-node key agreement, only four local nodes are communicated to each other.

For the communication overhead of the proposed protocol, the sensor nodes, ticket server, and key server collaborate with each other to produce an exclusive node-to-node key for each communication. However, with this process the computation of ticket and key generations are required at the ticket server and key server, respectively. According to this operation for each communication between nodes requires the key establishment process that will be perform at the ticket server and then key server. Let n be the number of sensor nodes. On the average there are  $\frac{n}{2}$  communications for node-to-node key generation. Each communication requires one ticket and one session key which means over all the computation of this method is approximately  $2\frac{2}{n}$ .

#### V. PERFORMANCE ANALYSIS AND DISCUSSION

The first question is how this protocol can provide freshness of the secure communication. The freshness of this protocol is from using the nonce numbers that are generate at the steps 1, 2, 5, and 6. When node A sends the request to node B, node A sends the request with the nonce number  $N_{A_1}$  and then node B communicates with the ticket granting server by sending its nonce number  $N_{B_1}$  along with the node's A nonce number. This is to guarantee that the communication is fresh and it is a newly established communication. When nodes A and B communicate with the key server S, nodes A and B regenerate the nonce numbers  $N_{A_2}$  and  $N_{B_2}$ . The nonce number regeneration is for the purpose to provide freshness to the communication with the key server S.

The second question is how this protocol can provide confidential. When the ticket granting server T communicates with node A, the ticket granting server uses node's A key,  $K_A$ , to encrypt the message. Only node A can decrypt the message. Similar to communicate with node B, the ticket granting server T uses node's B key,  $K_B$ , to encrypt the message. Only node B can decrypt the message. Then nodes A and B communicate with the key sever S, the message sent from nodes A and B are encrypted by the key server's keys,  $K_{TSA}$ , for the message sent by A and for the message sent by B; only the key server S can decrypt the message. Then the shared key,  $K_{AB}$ , is sent to nodes A and B being encrypted by node's A key, A, and node's A key, A, only nodes A and A can decrypt the message.

Finally, the last question is how this protocol can provide authentication. The protocol is authenticated by Message Authentication Code MAC which is used at steps 2 to 8. This cryptography uses a session key to detect both accidental and intentional modifications of the data. This can be achieved by using Cipher Block Chaining Message Authentication Code [?]. Two inputs, a message and a secret key that is known by only the originator its intended recipient(s), are required for a MAC authentication. The recipient can verify and authenticate the integrity of the message as only the sender has the shared secret key.

#### VI. CONCLUSION

The proposed method presents the node to node key agreement which is provided by the ticket granting server T and the key server S. There are several steps to set up a shared key for nodes A and B to securely communicate. Starting from node A sends a request to node B. This request is sent to node B along with the nounce number  $N_{A_1}$ . Node B receives the request from node A and asks for a ticket from the ticket granting server T. The request for the ticket is sent to ticket granting server T with nounce numbers  $N_{A_1}$  and  $N_{B_1}$ . This ticket is required for nodes A and B to communicate with the key server S. After the ticket is granted to nodes A and B, nodes A and B generate new nounce number  $N_{A_2}$  and  $N_{B_2}$ and send the new nounce numbers with the ticket to the key server S. The key server S issues the shared key to nodes Aand B. Finally nodes A and B can communicate with each other using the shared key.

The authentication, confidential, and freshness of the secured communication are efficiently achieved by this proposed method, as the proposed protocol uses the MAC to authenticate the messages, the shared key to communicate between nodes A and B, and the nonce numbers to refresh the messages.

In future work, we plan to focus on the key generation characteristics for the reason that a more efficient and scalable key generation can be achieved by considering the keys among the neighboring nodes. Additionally, further future work includes the problem of how the ticket can be generated at the ticket granting server T and how the shared key can be provided using criteria and the environment of the wireless sensor networks.

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# Secured Node-to-Node Key Agreement for Wireless Sensor Networks

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Abstract—Sensor networks are mostly deployed in unsecured environments, thus protecting a sensor network from any attack is critical in order to maintain the health of the network. Recently, many researchers have focused on making security for sensor networks available and reliable. In this paper, a secured node-to-node key agreement protocol is proposed to generate secured communication among principle nodes A and B, a ticket granting server, and a key server.

Since a sensor network is usually a resource-constrained infrastructure, it is not suitable for computationally expensive asymmetric key protocols such as public-private key cryptography. Therefore, setting up a shared key in our proposed protocol is based on a symmetric key protocol processed by two trusted agents, which are the ticket granting server and the key server. The data confidentiality, authentication, and freshness of the network security are also considered in the design of the proposed protocol.

Index Terms—Wireless sensor networks; Security; Authentication; Confidential; Freshness

## I. INTRODUCTION

Wireless sensor networks (WSN) have been identified as one of the most incredible technologies for many applications such as tank movement, ship arrivals and departures, etc. [1][2], [3]. For each area of wireless sensor networks, a cluster of sensor nodes is deployed to collect the data that is later reported to the network base station [4]. To collect the data in such a network, the sensor nodes can relay the data to the base station.

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Figure 1. The eight steps of the key agreement.

message authentication. They used the public key concept to achieve the broadcast authentication. Cryptographic techniques includes Merkle hash tree and identity-based signature scheme have been adopted to minimize the costs on computation and communication in wireless sensor networks.

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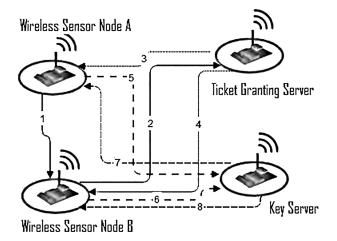


Figure 2. Illustration of the steps of the proposed node-to-node key agreement.

#### III. IMPLEMENTATION

In order for nodes to communicate in a wireless sensor network, a node-to-node key agreement has to be established. This key agreement provides security of the communication, which includes authentication, availability, and freshness. As mentioned in [11], due to the resource-constrained sensor nodes, the computationally expensive public-key cryptography is not suitable. Therefore, the protocol for symmetric-key is suggested.

For the purpose of gaining stronger freshness, the two trusted agents, key server S and the ticket granting server T, are included in our proposed protocol for the node-to-node key agreement. A message authentication code (MAC) is additionally used for two-party authentication and data integrity.

The following notations are used in this paper.

are principles, such as communication nodes  $N_{A_1}, N_{A_2}, N_{B_1}, N_{B_2}$  are Nonce numbers from A and B  $M_1 | M_2$ denotes the concatenation of message  $M_1$  and  $M_2$  $K_A$ is the key of node A shared with base station  $K_{AB}$ is the pairwise key shared between nodes A and  $K_{TSA}$ is the shared key between nodes T, S, and A $\{M\}_{k_{TSA}}$ is the encrypted message with the symmetric key  $T_n$ is the ticket number to communicate with T MACis the message authentication code

The proposed node-to-node key agreement provides secured key agreement and stronger key freshness. The steps of the key agreement are given in Fig. 1 and illustrated in Fig. 2.

The detail of each step of the proposed method is explained here. We start from step 1 when the wireless sensor node A would like to communicate with node B. Node A sends a request to node B with nonce number  $N_{A_1}$ . In step 2 node B sends  $N_{B_1}$  with a request for a ticket from ticket granting

server T. This ticket is used for communication between node A and B, and the key server S. In steps 3 and 4, the ticket granting server T grants a ticket and sends to nodes A and B.

The messages sent from the ticket granting server T to nodes A and B are encrypted with key  $K_A$  and key  $K_B$ , respectively. Then in steps 5 and 6, nodes A and B send the ticket to the key server S to ask for the shared key  $K_{AB}$ . At these steps, nodes A and B sends the messages to the key server with the nonce numbers  $N_{A_2}$  and  $N_{B_2}$  respectively. As mentioned earlier, the nonce numbers provide the freshness of the communication. The key server S receives the ticket and then issues the shared key  $K_{AB}$  to nodes A and B in steps 7 and 8.

For the confidentiality, all messages sent from either the ticket granting server T or the key server S to nodes A or B are encrypted by  $K_A$  or  $K_B$ . Thus only node A or node B can decrypt the messages. All these messages are authenticated with MAC protocol. Thus the proposed method provides the three properties of wireless sensor security for node-to-node key agreement; all of which are freshness, confidential, and authentication.

#### IV. CONCLUSION

The proposed method presents the node to node key agreement which is provided by the ticket granting server T and the key server S. There are several steps to set up a shared key for nodes A and B to securely communicate. Starting from node A sends a request to node B. Node B receives the request and asks for a ticket from the ticket granting server T. This ticket is required for nodes A and B to communicate with the key server S. After the ticket is granted to nodes A and B, nodes A and B send this ticket to the key server S. The key server S issues the shared key to nodes A and B. Finally nodes A and B can communicate with each other using the shared key.

The authentication, confidential, and freshness of the secured communication are efficiently achieved by this proposed method, as the proposed protocol uses the MAC to authenticate the messages, the shared key to communicate between nodes A and B, and the nonce numbers to refresh the messages.

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# **Solving Instability Problem in Soft Body Dynamics**

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**Keywords** – soft body, explicit Euler method, implicit Euler method, classical 4<sup>th</sup> order Runge Kutta method

#### **Abstract**

For computer simulation, a soft body has been used in many fields such as medical applications and entertainments. The soft body is composed of a list of surface points. Each surface point in the soft body has its own properties which of them are position, force, and velocity. In each time step, new position is computed by applying a new force using integration method such as explicit method or implicit method. In explicit Euler method is simple and fast in a term of computation. However, a disadvantage of this method is that the simulation creates instability problem and causes the soft body to disappear from the simulation scene when the force is too large. Thus, implicit method has been used to avoid this problem. Unfortunately, the implicit Euler method is time consuming since it requires high computation. In this paper, a classical 4th order Runge Kutta is adopted to efficiently implement the soft body models because it can solve instability problem and requires less computation compared to implicit Euler method. The results of the simulation show that the classical 4th order Runge Kutta method can avoid instability problem and can run faster than implicit Euler method.

#### 1. INTRODUCTION

Simulations of soft bodies such as cloths, hair, and liquid have main components which are set of surface points and movement of these points[1][2][3][4]. Each simulation time step, surface points are repositioned depending on the external or internal forces applying to each surface point. Due to computation of the external and internal forces, the simulation of soft bodies becomes more complicated than rigid objects especially in a term of stability if mass-spring system is selected to apply to each surfaces of the soft bodies.

Mass-spring systems have some drawbacks discussed in [5][6] where mass-spring systems may cause the instability problem when the time intervals are applied in

numerical method for the movement of the surface points. When large time intervals are applied to motion of soft bodies, the simulation creates instability problem that causes the soft bodies to disappear from the simulation scene. Because of this reason, the soft bodies are not able to maintain the original shape, and the forces can drag the soft body out of the scene.

For the movement of soft bodies using numerical methods, explicit and implicit Euler methods have been widely used. Explicit methods require small time intervals to solve for the differential equations of motion. However when the large time intervals are applied, instability problem occurs. The problem of instability in explicit methods have been mentioned in [5][6][7][8]. To solve this problem implicit methods [9] have been brought into an account. In particular, the implicit methods require more complex computation because the methods needs to use unknown values. In this paper, the classical 4th order Runge Kutta method is presented for the soft body simulation since it is simpler than implicit method and able to solve instability problem that occurs in explicit method. The results of the simulation show that the simulation using the method applied to the soft body are simple and can solve instability problem.

The paper is organized as follows: background of the soft body simulation, the methods applied in our soft body, implementation and results, and then conclusions and discussion of further work.

#### 2. BACKGROUND OF SOFT BODY

This section is divided into two subsections. The first subsection explains the mass-spring system for the deformation of the soft body surface and the second subsection reviews the numerical method to animate the soft bodies.

#### 2.1. Mass-spring system one the soft body surface

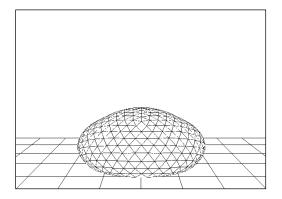


Figure 1. Mass-spring connections

Mass-spring system is used to connect the surface points in our soft body. Mass-spring force at surface point i that connects to surface point j can be calculated by applying spring constan,  $k_{ij}$ . In physic simulation, it is required that the springs don't oscillate forever but come to rest eventually. Thus, the dampers need to be combined to springs for realistic simulation. The equation of mass-spring force with spring-and damper at point i can be calculated as follows[5][6]:

$$\mathbf{F}_{si}^{t} = \sum_{\forall j | (i,j) \in E} k_{ij}(|x_{j} - x_{i}| - l_{ij}^{0}) \frac{(x_{j} - x_{i})}{|x_{i} - x_{i}|}$$

, where  ${\pmb F}_{si}^t$  is the net internal spring force exerted on point i for every point j in the neighborhood E,  $k_{ij}$  is the spring constant of the spring connecting points i and j,  $x_i$  and  $x_j$  are the positions of point i and j respectively,  $l_{ij}^0$  is the rest length of the spring between points i and j, and j is current time step. Since the spring applies equal and opposite forces to each connected point, it exerts  ${\pmb F}_{si}^t$  on one point and  $-{\pmb F}_{si}^t$  on the connected point.

$$\boldsymbol{F}_{di}^{t} = \sum_{\forall j \mid (i,j) \in E} k_{ij} h(\boldsymbol{v}_{i} - \boldsymbol{v}_{j})$$

, where  $\boldsymbol{F}_{di}$  is the damping force,  $k_{ij}$  is the damping constant between point i and j,  $\boldsymbol{v}_i$  is the velocity at point i,  $\boldsymbol{v}_j$  is the velocity at point j, t is current time step, and h is time interval.

The internal pressure force by assuming that there are gas molecules inside the soft body. Since pressure force is the vector sum of the pressure times area, point i can be modified to consider all surface areas and its neighboring connected points as follow:

$$\boldsymbol{F}_{pi}^{t} = \sum_{\forall jk | (i,j,k) \in E} a_{ijk} \hat{\boldsymbol{n}} \frac{1}{V} \boldsymbol{n} \boldsymbol{R} \boldsymbol{T}$$

, where  $\boldsymbol{F}_{pi}^{t}$  is the net pressure force experienced by point i at time t,  $a_{ijk}$  is surface area of the face connecting point i to all point pairs (i,k) in the neighborhood of i,  $\hat{\boldsymbol{n}}$  is a normal vector to surface where the pressure force is acting, V is volume of the body, n is gas mol number, R is the ideal gas constant, and T is the temperature of the body. We use Axis Aligned Bounding Box (AABB) to determine volume of the soft body.

#### 2.2. Numerical methods

To define the behavior of the deformation of the objects, each surface point has its own velocity, force, and position. The positions of surface points need to be approximated by numerical methods such as explicit Euler or implicit Euler methods etc. when the forces are applied on the soft bodies. Since Classical 4th order Runge-Kutta method is used in this paper, the detail of this method is described at the end of this subsection.

Explicit Euler Method

Let  $y = \phi(t)$  be the function of y' = f(t, y).

Assuming that  $t = t_n$ , we have

$$\phi' = f(t_n, \phi(t_n))$$

Using explicit Euler method, we obtain

$$\frac{\phi(t_{n+1}) - \phi(t_n)}{t_{n+1} - t_n} \cong f(t_n, \phi(t_n))$$

Replacing  $\phi(t_{n+1})$  and  $\phi(t_n)$  by the approximation values  $y_{n+1}$  and  $y_n$ , and replace  $(t_{n+1}-t_n)$  by h. Then solving for  $y_{n+1}$ , the explicit Euler's formula becomes:

$$y_{n+1} = y_n + f(t_n, y_n) \cdot h, \quad n = 0, 1, 2, \dots$$

Implicit Euler Method

Let  $y = \phi(t)$  be the function of y' = f(t, y).

Assuming that  $t = t_n$ , we have

$$\phi' = f(t_n, \phi(t_n))$$

Using implicit Euler method for  $\phi'$ , we have

$$\frac{\phi(t_n) - \phi(t_{n-1})}{t_n - t_{n-1}} \cong f(t_n, \phi(t_n))$$

Replacing  $\phi(t_n)$  and  $\phi(t_{n-1})$  by the approximation values  $y_n$  and  $y_{n-1}$ , and solving for  $y_n$ , the implicit Euler's formula

$$y_n = y_{n-1} + f(t_n, y_n) \cdot h$$
  
$$y_{n+1} = y_n + f(t_{n+1}, y_{n+1}) \cdot h$$

Note that this equation implicitly defines  $y_{n+1}$ , and must be solved in order to determine the value of  $y_{n+1}$ .

Classical 4<sup>th</sup> order Runge-Kutta method [10]

Let  $y = \phi(t)$  be the function of y' = f(t, y).

Assuming that  $t = t_n$ , we have

$$\phi' = f(t_n, \phi(t_n))$$

Using Classical 4<sup>th</sup> order Runge-Kutta method for  $\phi$ , we have

$$k_{1} = f(t_{n}, \phi(t_{n}))$$

$$k_{2} = f\left(t_{n} + \frac{h}{2}, \phi(t_{n}) + \frac{h}{2}k_{1}\right)$$

$$k_{3} = f\left(t_{n} + \frac{h}{2}, \phi(t_{n}) + \frac{h}{2}k_{2}\right)$$

$$k_{4} = f\left(t_{n} + h, \phi(t_{n}) + hk_{3}\right)$$

$$\phi(t_{n+1}) = \phi(t_{n}) + \frac{h}{6}(k_{1} + 2k_{2} + 2k_{3} + k_{4})$$

Replace  $\phi(t_{n+1})$  and  $\phi(t_n)$  by  $y_{n+1}$  and  $y_n$ , we have

$$y_{n+1} = y_n + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

#### 3. IMPLEMENTATION AND RESULTS

For the implementation using the classical  $4^{th}$  order Runge-Kutta method, the simulation calculates the spring force at the first step using spring constant, positions of surface points between the spring, and rest length of the spring. To ensure that the spring does not oscillate forever, the damping force is computed at the second step. Then the internal pressure force is introduced at the third step to provide the volume of the soft body. Fourth step combines all forces together. Then the new positions of the surface point is approximated using the classical 4th order Runge-Kutta method which means  $k_1, k_2, k_3$ , and

 $k_4$  are calculated at the fifth step for the computation of the velocity which is later computed at the sixth step. Finally each surface point is repositioned at the seventh step. The steps of simulation are presented as follows:

Step 1: Compute Spring force

$$\mathbf{F}_{si}^{t} = \sum_{\forall j | (i,j) \in E} k_{ij} (|x_{j} - x_{i}| - l_{ij}^{0}) \frac{(x_{j} - x_{i})}{|x_{j} - x_{i}|}$$

Step 2: Compute damping force

$$\boldsymbol{F}_{di}^{t} = \sum_{\forall j | (i,j) \in E} k_{ij} h(\boldsymbol{v}_{j}^{t} - \boldsymbol{v}_{i}^{t})$$

Step 3: Compute internal force

$$\boldsymbol{F}_{pi}^{t} = \sum_{\forall j | (i,j) \in E} a_{ij} \hat{\boldsymbol{n}} \frac{1}{V} nRT$$

Step 4: Combine all forces

$$\boldsymbol{F}_{i}^{t} = \boldsymbol{F}_{si}^{t} + \boldsymbol{F}_{di}^{t} + \boldsymbol{F}_{pi}^{t}$$

Step 5: Calculate  $k_1, k_2, k_3$ , and  $k_4$ 

Step 6: Compute a velocity of each surface point

$$\mathbf{v}_{i}^{t+h} = \mathbf{v}_{i}^{t} + \frac{h}{6}(k_{1} + 2k_{2} + 2k_{3} + k_{4})$$

Step 7: Computer a new position of each surface point

$$x_i^{t+h} = x_i^t + \boldsymbol{v}_i^{t+h} h$$

The result of simulation is presented in Table 1 which compares the performance of the numerical methods in the term of average frames per second (FPS). The simulation is set where the surface points freely move around the simulation scene. The average of each method is computed from the total of 10000 simulation frames and number of points are varied from 1000, 10000, 100000, and 1000000. From the average FPSs, the results show that the explicit Euler Method provides the best FPS, implicit Euler method presents the worst FPS, and the Classical 4<sup>th</sup> order Runge-Kutta method gives the FPS rate between explicit and implicit Euler methods. However explicit Euler method creates the instability problem when mass-spring systems are applied with large time intervals. The instability problem is presented in figures 2a) and 2b). Figure 2a) demonstrates the initial simulation scene with the soft body (towel) and the soft body (towel) disappears in figure 2b) because of the instability in the simulation scene.

Table 1. Comparison of three different numerical methods

	Numerical methods in motion (Average FPS)			
Number of points	Explicit Euler Method	Implicit Euler Method	Classical 4 <sup>th</sup> order Runge-Kutta method	
1000	327	298	301	
10000	287	256	275	
100000	255	189	221	
10000000	209	154	178	



**Figure 2a).** Initial simulation scene of the soft body (towel) with explicit Euler method



**Figure 2b).** Simulation scene of the soft body (towel) with explicit Euler method captured at frame 1000<sup>th</sup>

The simulation of the soft body (towel) with classical 4<sup>th</sup> order Runge-Kutta method is shown in figures 3a) and 3b). The Classical 4<sup>th</sup> order Runge-Kutta method has been applied to wrinkle effect on the soft body surface to present the motion of the soft body deformation. The results show that the classical 4<sup>th</sup> order Runge-Kutta method can successfully present the transitions of the soft body surface deformation.



**Figure 3a).** Initial simulation scene of the soft body (towel) with Classical 4<sup>th</sup> order Runge-Kutta method



**Figure 3b).** Wrinkle effect of the soft body (towel) surface with Classical 4<sup>th</sup> order Runge-Kutta method captured at frame 1000<sup>th</sup>.

## 4. CONCLUSION AND FUTURE WORK

In this paper, the classical 4th order Runge-Kutta method for the motion of soft body has been presented where the mass-spring system is applied to connect the surface points. Because large time intervals can create the instability problem in explicit Euler method which means the simulation is not able to successfully present the motion of soft body, the classical 4th order Runge-Kutta method has been chosen to solve this problem in this paper. There are seven steps of the implementation and the results of the simulation show that the classical 4<sup>th</sup> order

Runge-Kutta method can successfully present the motion and the deformation of the soft body.

For future works different applications such as the applications in medical or molecular fields can be considered for the simulation. Different bench marks for different models and scenarios should be implemented to compare the performance of the presented method.

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